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DEVELOPMENT OF THE MECHANICS OF SEDIMENT TRANSPORTATION

by

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With discussions by

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Introduction

The purpose of this paper is to outline briefly the history and development of the mechanics of sediment transportation and to indicate the information in this field that is available to the engineer and sedimentation specialist. A brief chronological history of the development will be given with some discussion of the more significant results. No attempt will be made to give a complete bibliography on this extensive subject since this has been done in an excellent fashion by a number of writers to which reference will be made. Finally, lines of research will be suggested which will furnish the information that must be obtained to help solve the many sediment problems now facing the people of our country.

The History of Sediment Transportation

Duboys and Gilbert. The science of sediment transportation was started by DuBoys 1/, who, in 1879, published the formula which now carries his name. He introduced the idea of the tractive force, or the shear force, at the bed of the stream. He also introduced the idea of the critical tractive force which most workers in this field define as the tractive force at which general movement takes place in the bed of the stream. The DuBoys formula (derivation given by Rouse 2/ is

$$G = \psi \tau_o \quad (\tau_o - \tau_c) \tag{1}$$

where G is the rate of sediment movement, in pounds per second per foot of width ψ is a coefficient, τ_o is the tractive force, or shear, at the bed and τ_c is the critical value of this force in pounds per square foot. The average shear at the bed is calculated by summing up the forces on a section of the stream one unit in length, as shown diagrammatically in Fig. 1. Assuming uniform flow, the pressure forces, F, at the two ends of the prism are the same and the only forces entering the equation are the weight of the water and the friction force along the bed. The average value of the friction force is

$$\bar{\tau}_o = \frac{\gamma AS}{p} = \gamma RS \tag{2}$$

where γ is the specific weight of water in lbs/cu.ft.

A is the cross sectional area of the stream in square feet

p is the wetted perimeter in feet

S is the slope in feet per ft, and
R is the hydraulic radius in feet

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1/ 2/ See "References" at close of comments.

The work of DuBoys has been followed by practically all workers on the bed-load problem, and reference can be found to his work in most publications in this field.

The next work of major importance in this field was that by G. K. Gilbert 3/, who did his work in about 1909 at the University of California and published it in 1914. Gilbert was one of the noted geologists of his time, and judging from his work on sediment transportation, he was an extremely able observer and research man. He pioneered in making sediment transportation experiments in flumes and many of his techniques are still being used. Although he did not develop a sediment transportation formula, he made some very comprehensive experiments, the data from which have been used over and over again. (Johnson 4/ has compiled all available bed-load data). Gilbert described very clearly the modes of movement. He observed that for incipient motion, the grains of sand are moved by rolling and sliding along the bed. As the velocity increases, he noted that the grains tend to take short jumps in curved orbits. He described this mode of transportation as saltation. As the flow was increased further, he noted that material was carried in suspension. He described in detail the sand waves similar to aeolian dunes which formed at the bed soon after incipient motion. As these dunes progress downstream the material is carried up the flat, upstream slope and deposited on the steep downstream slope. As the flow increases, the dunes are flattened and the sediment moves over the entire bed without any waves. When the velocity is increased further, another type of bed undulations appear, which he termed "anti-dunes." These dunes or sand waves move upstream and are periodic in nature, tending to form, build up to a maximum, and then disappear. They are accompanied by waves on the water surface similar in shape to the bed wave. Gilbert also introduced the term, "competence," which is applied to the characteristics of the stream for the conditions at which sediment commences to move. For instance, for a given slope and channel, the competent flow is that at which the bed material will begin to move.

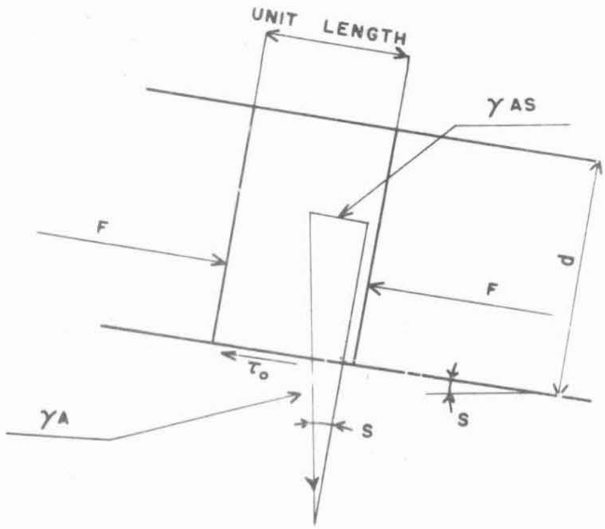

Fig. 1. Tractive Force on Channel Bottom.

Development Following World War I

Following World War I, the problem of transportation of material by streams was attacked vigorously both in Europe and in the United States. During this period also the movable bed hydraulic model was developed and much of the research on sediment transportation was in connection with developing techniques of representing alluvial streams to a small scale. One of the principal factors in the development of sediment transportation and hydraulic engineering in general in the United States was the establishment of travel fellowships by the late John R. Freeman. Through this means many promising young men studied in Europe where the science of hydraulics and the use of the model was developed much more than it was in this country. The inspiration obtained in this manner was responsible largely for the sediment studies in this country.

The result of this activity was the appearance of quite a few bed-load formulas of an empirical or semi-empirical nature, based on sediment transportation experiments in flumes. A number of the better known of these formulas are shown in Table 1. An inspection of these formulas will show that most of them are similar to the DuBoys formula and that they involve either a critical tractive force, or a critical flow. A study of the results of the various investigations will show that it is difficult to get a precise value of the critical tractive force or flow. The critical values are defined as the values existing when general movement first occurs. The term, "general movement," is difficult to define precisely and different observers are apt to have different interpretations of the term. The result is that the data of different workers are not always comparable. The fact that so many formulas appeared is very confusing. This confusion was clarified to a great extent by Johnson 5/, who compared a number of these formulas by plotting the same data according to a number of different formulas. He did this for data from

3/ 4/ 5/ See "References" at close of comments.

Table 1. Formulas for Rate of Bed-Load Transportation

Worker	Formula	Sediment	Eq. No.
U. S. Waterways Experiment Station (4)	$G = \frac{K}{n}(\tau_o - \tau_o)^m$	Sand mixtures	(3)
Chang (5)	$G = Kn\,\tau_o(\tau_o - \tau_o)$	Uniform sand	(4)
O'Brien (6)	$G = A\left(\frac{V}{R^{1/3}}\right)^m$	Sand mixtures	(5)
MacDougall (7)	$G = AS^m(Sq - Sq_o)$	Sand mixtures	(6)
Schoklitsch (8)	$G = \frac{A}{\sqrt{D}}\,S^{3/2}(q - q_o)$ $q_o = \frac{BD}{S^{4/3}}$	Uniform sand	(7)
Meyer-Peter (11)	$G = (BS\,q_1^{2/3} - B_1D)^{3/2}$	Uniform sand Large size	(8)

LEGEND

G = Rate of sediment movement in lbs./sec/ft of width

n = Mannings roughness coefficient

τ_o = tractive force at bed in lbs/sq.ft.

τ_o = critical tractive force at bed in lbs/sq.ft.

D = diameter of sand in mm.

S = slope of the energy gradient in feet per foot

q = rate of flow per unit width in cu. ft. per sec. per ft. of width

q_o = critical value of flow per unit width or flow when general movement of the bed material first starts - cu. ft. per sec. per ft. of width

q_1 = rate of flow per unit width in lbs per sec per ft of width

K, A, and B = empirical constants

m = empirical exponent

the U. S. Waterways Experiment Station 6/ for Formulas 1, 3, 4, 5, 6 and 8. Fig. 2 shows his graphs for Formulas 1, 3 and 4. It is very difficult from inspection to determine which of the formulas gives the best fit. By means of a statistical analysis of the various graphs, Johnson concluded that the goodness of fit for the three graphs shown in Fig. 2, as well as for graphs of equations 5, 6, and 8, was about the same. From this he concluded that the choice of equations could be made on the basis of the convenience in measuring the variables appearing in the formula.

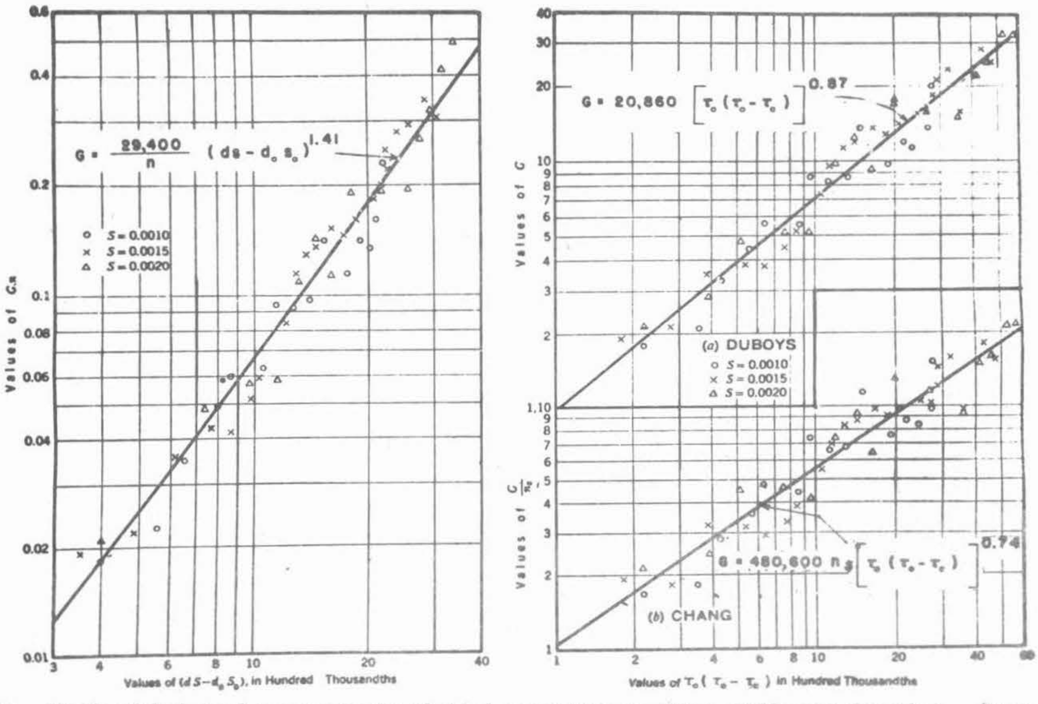

Fig. 2. Graph of bed-load measurements plotted according to three different formulas. (Reprinted from "Laboratory Investigations of Flume Traction and Transportation" (See "References," item 7, at close of comments)"

6/ See "References" at close of comments.

An inspection of the bed-load formulas in Table 1 will show that there are a number of constants in the equations which must be determined empirically before the equation can be applied. To begin with, the critical values of either the shear or the discharge must be determined. The difficulty in the definition of critical conditions makes it very difficult to determine these values and leads to inconsistencies between various workers. The same difficulties appear to apply to the determination of the other constants. The general impression that is obtained in looking at the formulas is that they are too simple to represent such a complicated system as a sediment-carrying stream. It is likely that some of the constants are actually functions.

As mentioned previously, much of the work on sediment transportation was done to develop techniques for movable bed model studies. The comprehensive work of Kramer 12/ and of the U. S. Waterways Experiment Station was done for this purpose. The objective was to find a sand mixture that could be used in a river model to simulate movement for all rates of flow occurring in the prototype. Kramer was one of the first workers to introduce the size frequency distribution or size grading of the sediment as a variable. He expressed the size grading empirically by the ratio of the areas above and below the 50 per cent line on the standard mechanical analysis graph where grain diameter is plotted against the percentage of the sample that is finer than the given diameter. He used this ratio in his expression for critical tractive force.

Newer Theories

Starting in about 1935, new approaches to the problem were developed by applying developments in basic fluid mechanics that appeared a few years before this time as a result of rapid developments in aeronautics. Shields 13/ showed that the tractive force for initial movement could be expressed as a function of the dimensionless parameter $\sqrt{\frac{\tau_o}{\rho}}\,D/\nu$ where D is the diameter of the sediment grains in feet, ρ is the mass density of the fluid in slugs per cubic and ν is the kinematic viscosity of the fluid in square ft/sec. This parameter is proportional to D/δ where δ is the thickness of the thin layer of laminar flow that occurs at the boundaries of a channel. The value of δ is given by the following formula derived by von Karman in 1930,

$$\delta = 11.6 \frac{\nu}{\sqrt{\frac{\tau_o}{\rho}}}$$

Fig. 3 is Shields' graph of dimensionless critical tractive force plotted against the parameter $\sqrt{\frac{\tau_o}{\rho}}\,D/\nu$ which is proportional to $\frac{D}{\delta}$. As this graph shows, he was also able to correlate the dune formation at the bed with the parameter - an extremely significant result, especially in view of the wide range of the variables which he covered. He varied the density of his sediments from 1.06 to 4.2 grams per cc and the diameter of his sediments from 0.36 to 3.4

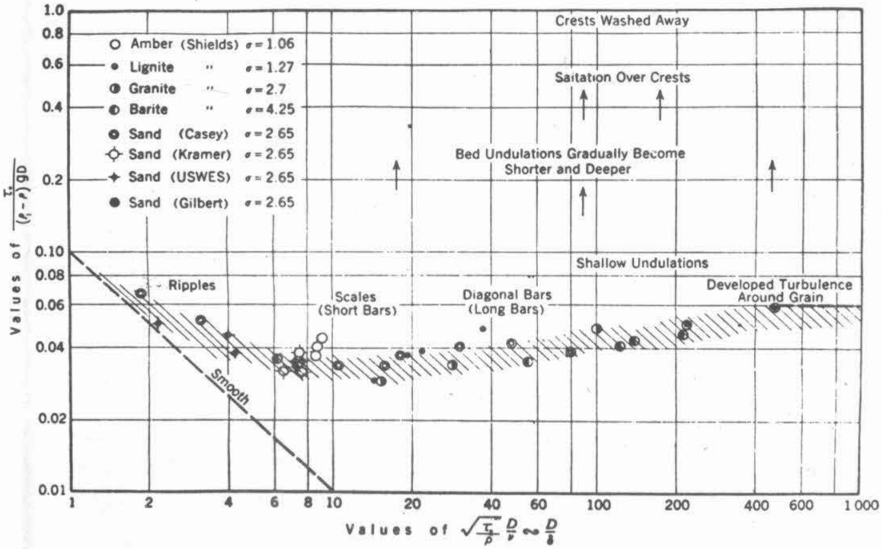

Fig. 3. Tractive force plotted against Reynolds number of sand grain according to Shields. (Reprinted from "Laboratory Investigations of Flume Traction and Transportation") (See "References," item 7, at close of comments.

12/ 13/ See "References" at close of comments mm. He classifies sand waves at the bed into ripples, short bars and long bars in decreasing order of height to length ratio, and shows in Fig. 3 that they can be classified according to D/δ. Shields states that any bed-load formula must contain parameters which will describe the bed configuration much as Fig. 3 does. He also develops an empirical bed-load formula which applies only to sediments which do not form ripples or high waves.

In 1941 H. A. Einstein 14/ published his ϕ - ψ formula for bed-load. It represented a radical departure from all of the previous bed-load formulas in that it was not based on the conception of a critical tractive force or critical condition of any kind. It is based on two main ideas. The first is that the sediment moves in steps or jumps and that the length of the step, L, is given by

$$L = \lambda D$$

where D is the diameter of the sediment grain and

λ is a coefficient

The second is that the number of particles that will move out of an area of unit width and length, L, in a second depends on the probability that the lift force will exceed the weight of the particle in water. By developing this idea with the use of dimensional analysis, he arrives at the formula

$$A \phi = f(B \psi) \tag{9}$$

where

$$\phi = \frac{1}{F} \frac{G}{p_1 g} \sqrt{\frac{p}{p_1 - p}} \frac{1}{g^{0.5} D^{1.5}}, \tag{10}$$

$$\psi = \frac{p_1 - p}{p} \frac{D}{RS}, \tag{11}$$

$$F = \sqrt{\frac{2}{3} + \frac{36 \mu^2}{g D^3 p (p_1 - p)}} - \sqrt{\frac{36 \mu^2}{g D^3 p (p_1 - p)}} \tag{12}$$

p_1 is the mass density of the sediment in slugs per cu.ft., g is the acceleration of gravity in ft/sec per second, μ is the coefficient of viscosity in pound seconds per sq.ft., and A and B are constants which are to be determined experimentally along with the function f. Fig. 4 is a graph showing data from bed-load experiments by various investigators plotted according to Einstein's formula. Fig. 4a, which is for uniform size material, follows a curve very well. For values of ϕ up to 0.4, the formula can be expressed as

$$0.465 \phi = e^{-0.391 \psi} \tag{13}$$

The fact that the results from various workers follow a curve is remarkable since the data cover experiments with material varying from 28.6 to 0.35 mm. in diameter, a variation of almost one hundredfold and for flows varying in depth from .06 to 3.6 feet. Fig. 4b is a graph of the data obtained by the U. S. Waterways Experiment Station on sand mixtures. The representative diameter of the sand mixtures was taken as that diameter for which 40 per cent of the material is finer or the sieve size through which 40 per cent of the material will pass. The data seem to follow a functional trend except that there is a range between curve (S) and (1) where the data do not fit a curve. Einstein attributed this to sorting in the experiments. However, the workers at the U. S. Waterways Experiment Station did not believe that this could be possible since they were extremely careful to avoid sorting. The result is that the discrepancy is not explained satisfactorily.

Einstein 14/ developed a method for determining the true shear on the bed by correcting for the friction on the walls. His formula and the Meyer-Peter formula, equation (8), are based on corrected values of R and Q, to compensate for the wall shear. The need for this was recognized by earlier workers and attempts to correct for it were made. Shoklitsch 10/ actually proposed a formula which is similar to Einstein's formula. Experimenters attempted to correct for the friction by using smooth walls so that the friction could be neglected, by roughening the walls with the same sand as in the bed and some collected the sediment only from the center portions of the flume, where the side wall effect could be neglected. Shields applied a wall shear correction in calculating values of τ

10/ 14/ See "References" at close of comments.

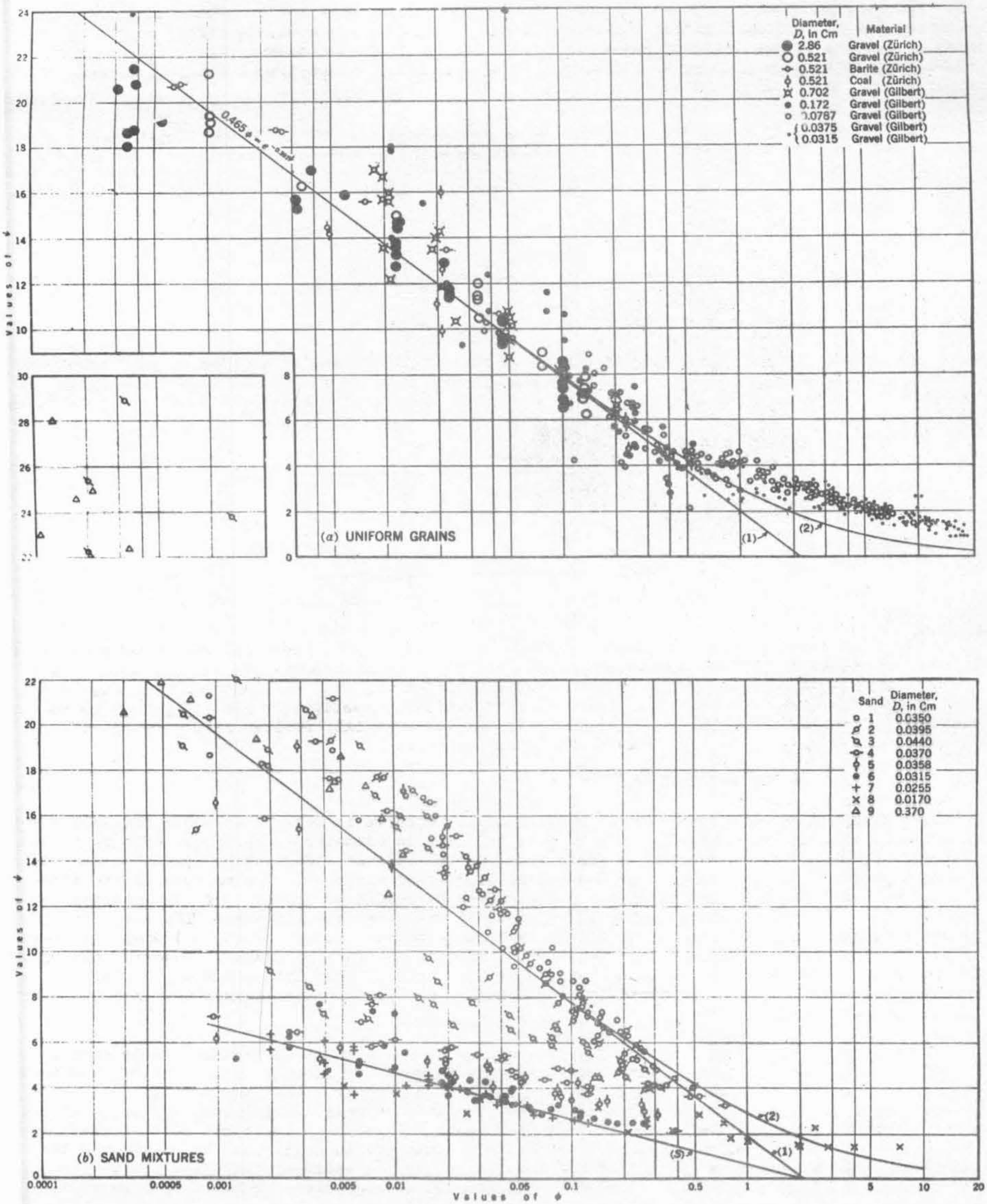

Fig. 4. Graph of Einstein ϕ - ψ bed-load formula. (a) for uniform sand, (b) for sand mixtures
(Reprinted from "Formulas for the Transportation of Bed-load")
(See "References," item 14, at close of comments)

Einstein 15/ carried his work farther by actually making sediment load measurements in two small streams. Mountain Creek, which is in South Carolina, is about 14 feet wide, has a slope of from .0014 to .0018 and bed material about 0.68 mm. in diameter. Measurements were made with flow depths up to about 1.4 feet. Goose Creek, in Mississippi, is about 13 feet wide, with a slope ranging from .0025 to .0031 and with material 0.25 mm. in diameter. Measurements were made in it with flow depths up to 0.6 foot. Fig. 5 shows a ϕ - ψ graph of the measurements on these streams. Curves 1 and 2 appearing in Fig. 4 have also been put on Fig. 5 for comparison. It can be seen that the field and laboratory measurements follow the same curve.

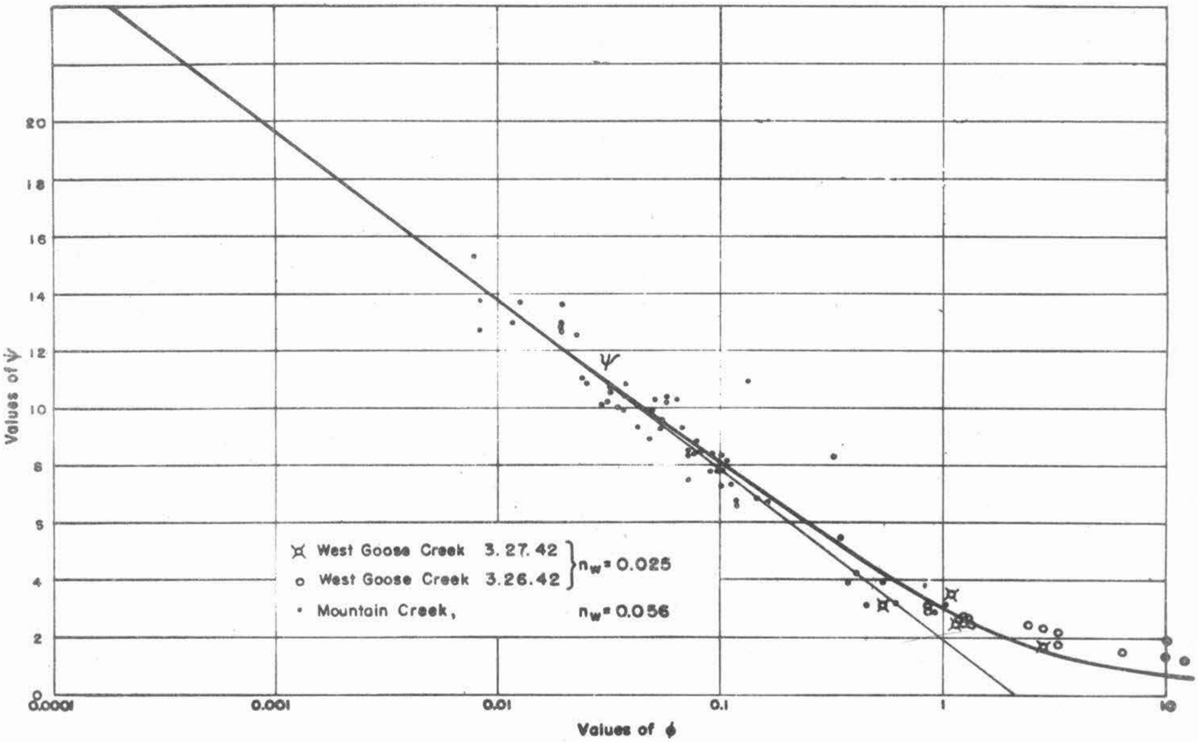

Fig. 5. Bed-load measurements in two streams plotted according to Einstein ϕ - ψ formula. (Reprinted from "Bed-load Transportation in Mountain Creek" (See "References," item 15, at close of comments)

In order to get a better idea of the size of streams covered by the ϕ - ψ graph, some numerical values will be substituted in the formula. By re-arranging the formula for ψ, the value of the hydraulic radius becomes

$$R = \frac{p_1 - \rho}{p} \frac{D}{\psi s} \tag{14}$$

If we now assume a ψ value of 4, quartz sand with diameters of 1 and 0.5 mm., and channel slopes of .001 and .0001, we obtain values of R shown in Table 2.

TABLE 2. VALUES OF HYDRAULIC RADIUS FOR ψ = 4

for assumed values of D and S

D (mm)	D (ft)	S	R (ft)
1.0	.00326	.001	1.3
		.0001	13
0.5	.00163	.001	0.65
		.0001	6.5

15/ See "References," at close of comments.

With 1 mm. sand and a slope of .001, the hydraulic radius for a value of $\psi = 4$ will be 1.3 feet. With the smaller slope this value will go up to 13 feet. This would be a condition for a relatively small stream, but would still cover many conditions of practical interest. By going to lower values of ψ and higher values of ϕ on the graph, conditions for still larger streams would be covered.

In order to use any bed-load formula, one must be able to calculate the hydraulic radius from the discharge, or conversely, one must be able to calculate the discharge from the hydraulic radius. This means that we must know the roughness of the channel. A number of experimenters have reported that the roughness of a sand bed varies with the rate of transportation. In Fig. 6 Einstein 16/ has plotted a factor Γ, against ϕ, where Γ is the ratio of the Manning roughness, n_s for no transportation to the roughness, n, at any given rate of transportation. The value of the roughness, n_s, is given by the Strickler formula

$$n_s = 0.0132\, D_1^{1/6} \tag{15}$$

where D_1 is a representative diameter of the sediment in mm. Einstein found that D_1 was the value for which 65 per cent of the material was smaller. With the above formulas, a complete calculation can be made of the bed-load transportation of a stream. It is the only complete bed-load equation available in the literature that is based on a modern analytical development. Kalinske 17/ has developed a formula based on statistical conceptions similar to those of Einstein. However, it involves quantities not measured in any experiments and its validity, therefore, remains to be proved. Recently Einstein 18/ has developed a method of calculating the load when an appreciable part of the sediment moves in suspension by dividing the load into bed and suspended load. Other workers 19/20/21/22/ have studied the forces acting on the sediment particles both experimentally and theoretically. Those were not intended to yield transportation formulas but to clarify the mechanism of movement which ultimately must form the basis for the formulas.

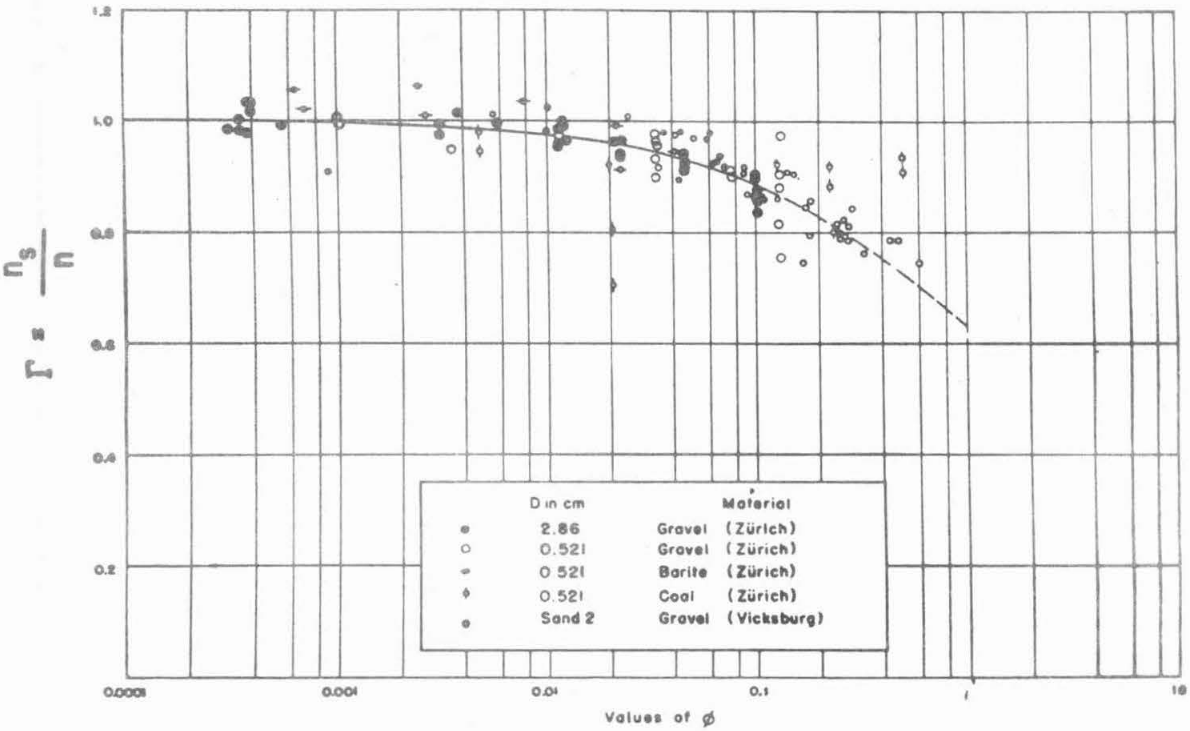

Fig. 6. Relative bed roughness plotted against ϕ according to Einstein. (Reprinted from "Flow on a Movable Bed" (See "References," item 16, at close of comments)

16/17/18/19/20/21/22/ See "References" at close of comments.

The very rapid development of the knowledge of transportation of suspended sediment by flowing fluids is attributable directly to the development in the general field of fluid mechanics which was stimulated principally by the development of aeronautics starting at about the time of World War I. The basic concept of turbulence suspension of sediment was advanced by Wilhelm Schmidt 23/ in 1925 for the suspension of dust in air. In 1933 Leighly 24/ applied this method to the suspension of sediment in streams. In 1933 O'Brien 25/ derived the differential equation for distribution of suspended load in a stream, which is

$$w c + \epsilon \frac{dc}{dy} = 0 \tag{16}$$

where w is the settling velocity of the particle in the still fluid, c is the concentration at a distance, y from the bottom, and ϵ is the so-called "exchange coefficient" which has the dimension of velocity times length. The left-hand term of the equation is merely the mass rate of settling of the particles under the force of gravity. The right-hand term is the rate of upward movement due to turbulence mixing. This work of O'Brien's set off a rapid sequence of developments which, in a period of a relatively few years, has developed the knowledge of suspended material to a degree comparable, if not in excess of that attained by the bed-load work in a period of about 60 years. O'Brien pointed out that the quantity, ϵ can be determined for momentum transfer from the velocity distribution by the formula,

$$\tau = p \epsilon \frac{dV}{dy} \tag{17}$$

where τ is the shear stress and V is the velocity at a distance y from the bottom. If ϵ is constant then by integrating equation (16) we get the equation

$$C = C_a e^{-\frac{w}{\epsilon}(y - a)} \tag{18}$$

where C_a is the concentration at some arbitrary level distant, a, from the bottom. This equation was checked by Hurst 26/ and Rouse 27/ for steady conditions with artifically-produced turbulence which was uniform over the depth, thus giving the condition of ϵ = constant. Dobbins 28/ has checked the relation for unsteady conditions with uniform turbulence. By assuming the von Karman logarithmic velocity distribution law,

$$\frac{V - V_{max}}{\sqrt{\frac{\tau_o}{\rho}}} = \frac{2.3}{k} \log \frac{y}{d} \tag{19}$$

ϵ can be calculated from eq. (17) and eq. (16) can be integrated to give the sediment distribution in a 2-dimensional uniform flow of depth d. The result of this integration is given by

$$\frac{C}{C_a} = \left[\frac{d-y}{y} \times \frac{a}{d-a}\right]^z \tag{20}$$

where $$z = \frac{w}{k\sqrt{gdS}}$$

k is the von Karman universal constant for turbulent flow which is about 0.4 for clear fluids, g is the acceleration of gravity, and S is the slope of the stream.

This equation was first published by Rouse 29/ in 1936. Fig. 7 is a graph of this equation for a number of assumed values of z. It will be noted that as z becomes small, the concentration tends to become more nearly uniform, while for high values of z the concentration vanishes in the upper part of the stream and most of the material is carried near the bed. It must be pointed out that this equation assumes that the transfer coefficient, ϵ, for momentum is the same as that for sediment. This is not necessarily so, as experiments have shown. Equation (16) was checked by Christiansen 30/ by applying it to field measurements of suspended load in the Imperial Valley Irrigation Canals. The form of the equation was checked by Vanoni 31/ in the laboratory experiments. The results of some of these experiments are shown in Fig. 8, in which measured concentration is plotted against depth. The curve at the right is for uniform material of 0.1 mm. diameter and the measurements follow a curve with value of the exponent z_1 of 0.34. The value of the exponent calculated from the formula is 0.423 and gives the dotted curve in Fig. 8.

23/24/25/26/27/28/29/30/ See "References" at close of comments.

In this case the measured concentration was actually more uniform than the concentrations predicted from the theory. (A similar result was found by Anderson 32/ for distribution of suspended load in a natural stream). The curve at the left is for a uniform material of 0.16 mm. diameter, and in this case, the value of the measured and calculated exponent agree very well. These results show the trend found in the experiments and indicate that although the form of the equation is correct, the constants given by the theory are in error.

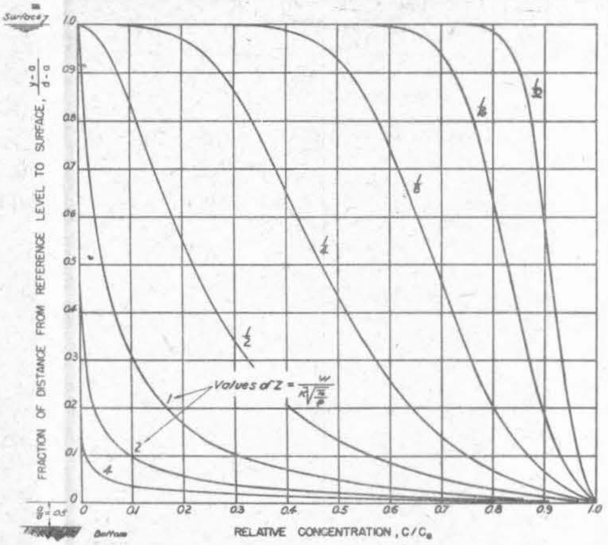

Fig. 7. Graph of theoretical suspended load distribution equation No. 20.

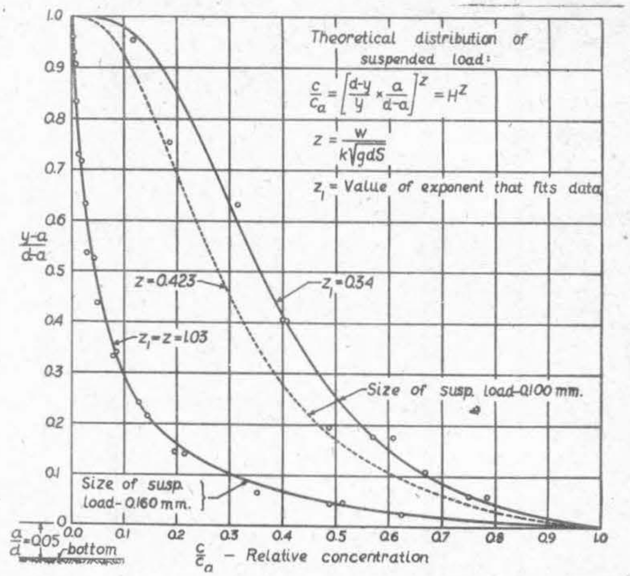

Fig. 8 - Comparison of theoretical and measured suspended load distribution for two sizes of sediment.

A very significant observation during the experiments was that sediment in suspension tended to cause an increase in the average velocity. This fact was attributed to the reduction in intensity of the turbulence by the suspended sediment. The energy to support the sediment must come from the turbulence and by giving up its energy the turbulence is less effective in mixing or agitating the flow. This is the same as saying that the transfer coefficient ϵ is reduced. From equation (17) it follows that if τ is kept constant as the sediment load increases, the value of the velocity gradient, and hence of the velocity, must increase. Careful velocity distribution measurements confirmed this. As the load increased, the slope of the velocity distribution curve also increased, tending to give greater differences in velocity between the bottom and the top. The amount of this increase can be expressed by the factor, k, in equation (19), since it was found that the velocity distribution equation even for relatively high loads still kept the logarithmic form of equation (19). Another factor that was found to vary with the amount of the load was the exponent, z. Fig. 9 is a graph of the three quantities, z, k and the resulting roughness coefficient, n, plotted as functions of the mean sediment concentration for 0.1 mm. sand. It will be seen that the roughness coefficient and the quantity, k, both are reduced as the concentration increases. The value of the exponent, on the other hand, increases as the sediment concentration increases. It will be remembered from Fig. 7 that as z increases, the distribution of material in the flow tends to become less uniform. Fig. 9, then, indicates that as the load increases, the sediment behaves as a coarser sediment, or, in other words, the ability of the stream to support it is lessened.

Another series of pertinent observations was that of the behavior of sediment in the flume cross section as a whole. The material tended to travel in bands or ribbons rather than uniformly, leading one to believe that there were certain secondary circulations present. No detail study was made of these, but there is some evidence indicating that it may be an important factor in the phenomenon. If so, the problem is much more complicated than had been anticipated since the

32/ See "References" at close of comments.

shape of the channel and the distribution of roughness over the wetted perimeter must certainly be important factors in the determination of secondary circulations.

It must be pointed out that the sediment distribution, equation (20), gives only the relative concentration and cannot be used to determine the load unless the concentration, C_a, at some level is known. Lane and Kalinske 33/ have developed an expression for the concentration just above the bed in terms of the properties of the sediment at the bed and a normal distribution of turbulence fluctuations. This is a step in the direction of calculating the total load of a stream in terms of the hydraulic factors. The trend in recent years has been to make less and less distinction between the material carried near the bed and that carried in suspension. In the opinion of the writer, this is a very sound trend since there does not appear to be any analytical reason for dividing the sediment transportation of a stream into the bed-load and the suspended load. The work of Einstein, Anderson, and Johnson 34/ in classifying the load of a stream into the "washload" and the "bed material load" has done much to bring this about. These writers defined the washload as a very fine material that is carried in suspension and that does not appear in the bed in appreciable quantities. They pointed out that this material is usually not a function of the stage, but depends on other conditions, such as the distribution of rainfall, condition of the watershed, etc. The bed material is that portion of the load having the same composition as the material in the bed. The rate of transportation of this material is a function of the stage.

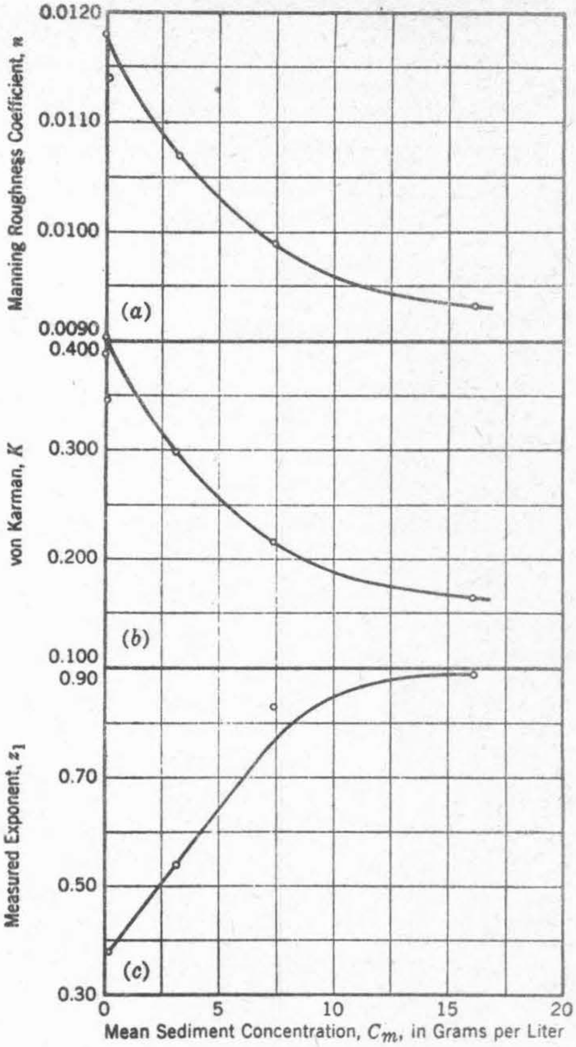

Fig. 9. Variation of apparent roughness , von Karman k and exponent Z_1 in sediment distribution equation with mean concentration. (Reprinted from "Transportation of Suspended Sediment by Water") (See "References," item 31, at close of comments).

Conclusions

One of the most important questions to be answered by a discussion of this kind is "Do we have a sediment transportation formula that can be applied to natural streams?" The only formula that is complete enough to be considered in such a discussion is the formula of Einstein. It agrees with a wide range of laboratory experiments from a number of sources and has also been found to agree with field measurements in small streams. The range covered by the formula up to $\phi = 10$ does not cover conditions where the rates of transportation are relatively high, such as in many of our streams during flood stage. This formula has not actually been tried on larger streams and therefore has not proved itself completely. However, it shows good promise and can at least serve to guide engineers in planning works where sediment transportation is involved. Formulas of the DuBoys type apparently fit only a very narrow range of conditions. They can probably be used as interpolation formulas on particular streams provided one can determine the constants by a combination of laboratory and field measurements. However, it does not seem possible to use this type of formula to calculate sediment loads without making elaborate field checks.

33/ 34/ See "References" at close of comments.

It is clear from this discussion that the sediment transportation problem is far from being solved. The fact that many able investigators have devoted much time to it without making more progress is evidence that the problem is complicated. Therefore, it is one that requires continued undivided effort on the part of the able investigators if we are to progress with it. The economic importance of the problem needs no emphasis. This has already been done by the elaborate programs of development on our major streams involving the expenditure of many millions of dollars. If we are to do a creditable job on this development, then we must learn more about sediment transportation and the mechanics of streams in general.

The following is a list of the problems on sediment transportation that I feel are of importance and should be studied further.

1. The laws governing the transportation of natural sand mixtures, such as found in streams.

2. The factors governing the production of sand waves and their effect on the roughness of the channel and the flow characteristics.

3. Detailed studies of the turbulence distribution and the effect of the sediment on the turbulence.

4. Studies of the exchange coefficient for suspended sediment and the factors governing its variation.

5. Studies of secondary circulation and its influence on sediment transportation and stream behavior.

6. Measurements in natural streams, to assist in extending the reliable range of equations for sediment transportation.

As pointed out previously, such investigations must involve continued effort on the part of highly trained research men. I estimate that there are less than 10 professional men in this country devoting a major part of their time to the study of the mechanics of sediment transportation. This is far too small a number. In my estimation, the sediment problem certainly ranks in importance with the more popular new problems of atomic energy, rocket propulsion, and travel at supersonic speeds. Yet it is receiving a negligible amount of research effort compared to these problems. It is up to those of us that know the vital importance of the sediment problem to call it to the attention of the people so its study will receive adequate financial support and the attention of our many research engineers and scientists.

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DISCUSSION

M. A. MASON,* This summary of the development of our knowledge of the mechanics of sediment transportation needs no amplification of its technical aspects, but its philosophical aspects can well be considered.

Recall that DuBoys classic work was the outgrowth of a study of natural river problems, specifically a question of the quantity of material carried by the Rhone River. Today we find that despite the advances made since DuBoys, the same question cannot be answered without reservation. Dr. Vanoni attributes this condition in part to the complexity of the problem, and in part to the small number of competent workers in the field. I believe a third factor, presently of transcendent importance, should be considered. That factor is our apparent inability to relate the fundamental technical aspects of the problem to our common economy in such a way that the financial support required to prosecute needed research and studies is available and assured.

We, as investigators or technical administrators, are all familiar with this phase of the problem. We are all in accord, I believe, that the technical problem involved could be solved if adequate financial support for a well-conceived overall program of study were assured. But have we prepared such a well-conceived overall program? Have we thoroughly examined, even in our own minds, the justification for the expenditure of funds, either private or public, on such a program? We must admit negative replies to these questions - we must admit, also, our frequent inability to convince even our own departmental budget and administrative officials of the necessity for our own limited programs. This is a deficiency in the leadership that can be reasonably expected of engineers. It is a deficiency that can be supplied by the Inter-Agency

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River Basin Committee, by the professional societies, even by a group of workers in the field. Certainly it is a deficiency that, under present conditions, is a major contributing cause to the difficulty of further work in this field.

The formulation of a well-conceived program in sediment transportation research is by no means a simple task. Effort in the past, with a few notable exceptions, has been concentrated on the practical aspects of the river and stream, bed and suspended load transportation problems; but the whole field is much broader. The list furnished by Dr. Vanoni has not exhausted his own possibilities, and each of us could add fundamental items. In the study of shore phenomena, for example, sediment, or more accurately, material, transportation is a major concern. Sediment is carried by streams to the sea as an ultimate destination; arriving there it is subjected to new transportation processes. Littoral movement by water forces and aeolian transportation are the more important processes. In each of these the relatively simple conditions of stream transportation are complicated enormously by the variety and nature of the forms of energy active at the shore. Only in a shore environment can we find material movement governed by the extreme violence of a breaking wave associated intimately with the turbulent upsurge and super-critical return flow down the beach of the same wave. Superimpose on this picture littoral currents resulting from tidal or wind effects and the composition is complicated indeed. Yet this is a legitimate and economically important sediment transportation problem. At the other extreme we can consider the transportation effects of ordinary rainfall on a vegetated area. Wind transportation phenomena must be included in our program, the change in fluid medium from water to air being a legitimate variation that our finally developed mechanics knowledge should be capable of handling.

Is it not reasonable to think of the common fundamental aspects of each of these varied yet related problems gathered into a program of basic study and research jointly financed and prosecuted? With this basic data in hand will not the long periods of comparative sterility between DuBoys, Gilbert, and Einstein be avoided? Will there not be opportunity for a more logical and successful appeal for financial support from government, university, and industrial funds?

Perhaps this latter question involves a realism that has no place in a technical discussion. But the record presented by Dr. Vanoni indicated to my mind that the deficiencies in our knowledge of the mechanics of sediment transportation could be attributed more to lack of specification of what we need to know and <u>of support in satisfying our needs</u> than to lack of technical competence in solving the problems once adequately defined. If this is true then further advances by other than a system of chance are contingent on supplying these past and present deficiencies. To my mind this can be done best and only by a wise, realistic recognition that what we have been doing is not enough, and by translating that recognition into a deliberate effort to actively endow critically needed basic research.

PARKER D. TRASK.* Mr. Vanoni's paper is on a subject of fundamental importance to everybody who works in the field of sedimentation. We cannot hope to solve most of our problems until we know the basic laws governing the erosion, transport, and deposition of sediments. Most of us are concerned with practical applications of these laws and we make the best approach we can by means of empirical data; but if we knew how the fundamental factors behaved and the inter-relationships of these factors one with another, we could predict what will happen with much greater certainty than we can now.

Messrs. Vanoni, Einstein, Lane, Johnson, Sverdrup, Monck, and the handful of others who focus their attention on these natural laws should be encouraged to continue their work, not only with funds and facilities, but also by the participation of others in this field.

H. A. EINSTEIN.** I thought you might be interested to know, in a general way at least, in which direction research in this field seems to move. Maybe I can make a few remarks about that. You have seen from Dr. Vanoni's paper that there exist essentially two separate theories: one for bed load and one for suspended load. The bed-load theory tries to give the rate at which sediment is transported under certain given flow conditions. The suspended-load theory tries to do something entirely different. It doesn't give the rate itself but it gives the distribution of concentrations as ratios over the different locations in the flow. It has been found and brought out, especially by Professor Lane, who unfortunately isn't here, that also for suspended loads

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there exist in the river certain laws according to which, at least for the coarser particles, the absolute value of the concentration is a function of the flow. If such a relationship exists in nature then we must be able to find why it exists and must be able to predict it. It has been in this direction that the latest endeavors have pointed. I can't describe here in detail how that was done, but the approach is based on the general idea that the rate at which sediment moves right at the bed can be given under all conditions by what we usually call a bed load formula. This bed load moving right at the bed in a layer about 2 grain diameters thick represents a sediment concentration right there near the bed from where we are able then to calculate the concentrations higher up by means of the suspended-load theory. In other words, the two types of approach can be combined with comparative ease into one unit.

A big difficulty that we still encounter in trying to predict the transport in rivers is based on the fact that we do not know enough about the behavior of different sized particles within a mixture. It has been possible in many cases to assume that the different particles behave very much alike or, in other words, that we find the same mixture in the bed and in movement. As soon as we combine into one unit the two theories of bed load and suspended load that assumption doesn't hold at all any more, and then we must replace it by another one. The new assumption stipulates that the movement of any one particle is practically independent from the movement of the other particles and may be fully described as a function of the flow. Now, I can't very well elaborate on the way this assumption works out, but it is a very reasonable one and seems to give good results. I don't think that it will be very long until we are able to really calculate the transport of material which moves partly in suspension and partly as bed load, which is the normal case in most of our Western streams. I believe if we do as the other speakers mentioned, that is, if we try to impress those agencies and people who are responsible for the distribution of funds and induce them to spend just a small part of the total research funds on this problem, we should be able pretty soon to solve a great number of practical problems by means of a theoretical approach.